

# Quantum Field Theoretic Treatment of the Non-Forward Compton Amplitude in the Generalized Bjorken Region

Johannes Blümlein <sup>a\*</sup> Bodo Geyer, Markus Lazar<sup>b</sup> and Dieter Robaschik<sup>c</sup>

<sup>a</sup>Deutsches Elektronen-Synchrotron, DESY, Platanenallee 6, D-15735 Zeuthen, Germany

<sup>b</sup> Institut für Theoretische Physik, Universität Leipzig, Augustusplatz 10, D 04109 Leipzig, Germany

<sup>c</sup>Institut für Theoretische Physik, Karl-Franzens-Universität Graz, Universitätsplatz 5, A-8010 Graz, Austria

A quantum field theoretic treatment of the leading light-cone part of the virtual Compton amplitude is presented. The twist-decomposition of the operators is performed by a group-theoretic procedure respecting the Lorentz group  $O(3,1)$ . The twist-2 contributions to the Compton amplitude are calculated and it is shown that the electromagnetic current is conserved for these terms. Relations between the amplitude functions associated to the symmetric and asymmetric part of the Compton amplitude are derived. These relations generalize the Callan-Gross and Wandzura-Wilczek relations of forward scattering for the non-forward Compton amplitude.

## 1. INTRODUCTION

Compton scattering of a virtual photon off a hadron  $\gamma_1^* + p_1 \rightarrow \gamma_2^* + p_2$  is an important processes in QCD: theoretically it can be treated in detail and experimentally it can be tested for a large variety of processes. In lowest approximation in the electromagnetic coupling it is described by

$$T_{\mu\nu}(p_+, p_-, q) = i \int d^4x e^{iqx} \times \quad (1.1)$$

$$\langle p_2, S_2 | T(J_\mu(x/2) J_\nu(-x/2)) | p_1, S_1 \rangle ,$$

where

$$p_\pm = p_2 \pm p_1, \quad q = \frac{1}{2}(q_1 + q_2),$$

with  $q_1$  ( $q_2$ ) and  $p_1$  ( $p_2$ ) being the four-momenta of the incoming (outgoing) photon and hadron, respectively, and  $S_1, S_2$  being the spins of the initial- and final-state hadron, where  $p_1 + q_1 = p_2 + q_2$ . The generalized Bjorken region is the asymptotic domain being defined by

$$\nu = qp_+ \longrightarrow \infty, \quad -q^2 \longrightarrow \infty ,$$

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where the two scaling variables

$$\xi = -\frac{q^2}{qp_+}, \quad \eta = \frac{qp_-}{qp_+} = \frac{q_1^2 - q_2^2}{2\nu} \quad (1.2)$$

are fixed. Of special experimental importance are the cases of deep inelastic scattering (DIS) described by the absorptive part of the forward Compton amplitude,  $\eta = 0$ , and the deeply virtual Compton scattering (DVCS) with one real outgoing photon  $q_2^2 = 0$  corresponding to  $\xi = -\eta$ . In the generalized Bjorken region the amplitude (1.1) is dominated by the light-cone singularities which allows to apply the (non-local) operator product expansion of  $T(J_\mu(x/2) J_\nu(-x/2))$ .

Our aim is a detailed quantum field theoretic investigation of that approach, cf. Refs. [ 1, 2]. In contrast to earlier considerations [ 3, 4] we take into account the explicit twist decomposition of the *non-local vector operators*, and their matrix elements, which thereby occur. The twist decomposition in the case of the quark-antiquark operators has been treated in [ 5]. An extension to the gluon operators and more general multi-particle operators is given in [ 6]. The relations between vector and scalar operators of twist 2 allows to express the final results with the help of matrix elements of the scalar operators only. In addition, this procedure leads to the deriva-

tion of new relations [ 2] on the amplitude level which correspond in the case of forward scattering to the Callan-Gross and Wandzura-Wilczek relations. Furthermore, the electromagnetic current conservation may be shown to hold on the level of the twist 2 contributions.

## 2. LIGHT-CONE-EXPANSION

The Compton amplitude for the case of non-forward scattering, Eq. (1.1), in the generalized Bjorken region is dominated by the light-cone singularities. Therefore, the  $T$ -product of the electromagnetic currents will be approximated by its non-local light-cone expansion [ 7] – a summed-up form of the local light-cone expansion which allows for a quite compact representation of the resulting expressions.

Let us present a series of intuitive arguments leading to that approximation. We start from the renormalized time-ordered operator product ( $S$  being the renormalized  $S$ -matrix):

$$\hat{T}_{\mu\nu}(x) = iRT [J_\mu(x/2) J_\nu(-x/2) S] .$$

At first we consider this expression in the Born approximation

$$\hat{T}_{\mu\nu}(x) = -e^2 \frac{x^\lambda}{2\pi^2(x^2 - i\epsilon)^2} \times \left[ \bar{\psi}\left(\frac{x}{2}\right) \gamma_\mu \gamma_\lambda \gamma_\nu \psi\left(\frac{-x}{2}\right) - \bar{\psi}\left(\frac{-x}{2}\right) \gamma_\nu \gamma_\lambda \gamma_\mu \psi\left(\frac{x}{2}\right) \right] .$$

Here  $e$  denotes the charge of the fermion field  $\psi$ . We dropped the flavor indices in the expressions considered. Reordering in the standard way the Dirac-structure we obtain

$$\hat{T}_{\mu\nu}(x) = -e^2 \frac{x^\lambda}{i\pi^2(x^2 - i\epsilon)^2} \times \left[ S_{\alpha\mu\lambda\nu} O^\alpha\left(\frac{x}{2}, \frac{-x}{2}\right) - i\varepsilon_{\alpha\mu\lambda\nu} O_5^\alpha\left(\frac{x}{2}, \frac{-x}{2}\right) \right] , \quad (2.1)$$

where

$$S_{\alpha\mu\lambda\nu} = g_{\alpha\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\alpha\nu} - g_{\mu\nu}g_{\lambda\alpha} .$$

The essential objects are the bilocal operators

$$O^\alpha\left(\frac{x}{2}, \frac{-x}{2}\right) = \frac{i}{2} \left[ \bar{\psi}\left(\frac{x}{2}\right) \gamma^\alpha \psi\left(\frac{-x}{2}\right) - \bar{\psi}\left(\frac{-x}{2}\right) \gamma^\alpha \psi\left(\frac{x}{2}\right) \right] , \quad (2.2)$$

$$O_5^\alpha\left(\frac{x}{2}, \frac{-x}{2}\right) = \frac{i}{2} \left[ \bar{\psi}\left(\frac{x}{2}\right) \gamma_5 \gamma^\alpha \psi\left(\frac{-x}{2}\right) + \bar{\psi}\left(\frac{-x}{2}\right) \gamma_5 \gamma^\alpha \psi\left(\frac{x}{2}\right) \right] . \quad (2.3)$$

Expression (2.1) satisfies electromagnetic current conservation in the case of free fields, i.e. at zeroth order in QCD. We are interested, however, in the case of general fields  $\psi$  and the twist-2 operators associated to them. We calculate these operators at leading order passing the following steps.

STEP 1: Use gauge invariant operators in place of (2.2), (2.3). This is achieved by including the phase factor  $U(y, z) = \mathcal{P} \exp(ig \int_z^y A_\mu dx^\mu)$ . The integration can be performed over a straight path connecting  $y$  and  $z$ .

STEP 2: Perform the twist decomposition of these operators according to [ 5] and restrict to the twist-2 (axial) vector operators only.

STEP 3: Take the operators as renormalized ones (at the light-cone) to all orders of QCD:

$$O_{\alpha}^{\text{tw}2}\left(\frac{\tilde{x}}{2}, \frac{-\tilde{x}}{2}\right) = \frac{i}{2} RT \left\{ \left[ \bar{\psi}\left(\frac{\tilde{x}}{2}\right) \gamma_\alpha U\left(\frac{\tilde{x}}{2}, \frac{-\tilde{x}}{2}\right) \psi\left(\frac{-\tilde{x}}{2}\right) - \bar{\psi}\left(\frac{-\tilde{x}}{2}\right) \gamma_\alpha U\left(\frac{-\tilde{x}}{2}, \frac{\tilde{x}}{2}\right) \psi\left(\frac{\tilde{x}}{2}\right) \right]^{\text{tw}2} S \right\} , \quad (2.4)$$

$$O_{5\alpha}^{\text{tw}2}\left(\frac{\tilde{x}}{2}, \frac{-\tilde{x}}{2}\right) = \frac{i}{2} RT \left\{ \left[ \bar{\psi}\left(\frac{\tilde{x}}{2}\right) \gamma_5 \gamma_\alpha U\left(\frac{\tilde{x}}{2}, \frac{-\tilde{x}}{2}\right) \psi\left(\frac{-\tilde{x}}{2}\right) + \bar{\psi}\left(\frac{-\tilde{x}}{2}\right) \gamma_5 \gamma_\alpha U\left(\frac{-\tilde{x}}{2}, \frac{\tilde{x}}{2}\right) \psi\left(\frac{\tilde{x}}{2}\right) \right]^{\text{tw}2} S \right\} , \quad (2.5)$$

where

$$\tilde{x} = x + \zeta [\sqrt{(x\zeta)^2 - x^2\zeta^2} - (x\zeta)] / \zeta^2$$

and  $\zeta$  is a subsidiary vector. Therefore our final expression to be considered in the following reads

$$\hat{T}_{\mu\nu}^{\text{tw}2}(x) = -e^2 \frac{\tilde{x}^\lambda}{i\pi^2(x^2 - i\epsilon)^2} \times \left[ S_{\alpha\mu\lambda\nu} O_{\alpha}^{\text{tw}2}\left(\frac{\tilde{x}}{2}, \frac{-\tilde{x}}{2}\right) - i\varepsilon_{\alpha\mu\lambda\nu} O_{5\alpha}^{\text{tw}2}\left(\frac{\tilde{x}}{2}, \frac{-\tilde{x}}{2}\right) \right] . \quad (2.6)$$

## 3. TWIST DECOMPOSITION

We are confronted now with the twist decomposition of non-local operators. The concept of twist was originally introduced in [ 8] and successfully applied in [ 9]. The twist decomposition of simple non-local operators was studied in

[ 10]. for the first time. A unique group theoretical procedure, based on the decomposition of related *local* tensor operators into irreducible ones with respect to the Lorentz group  $O(3,1)$  has been introduced recently and successfully applied to non-local tensor operators up to second rank [ 5, 6].

As an example let us consider the following (uncentered, unsymmetrized) quark operator

$$O_{\Gamma}(0, \kappa x) = [\bar{\psi}(0)\Gamma U(0, \kappa x)\psi(\kappa x)] ,$$

where  $\Gamma = \{1, \gamma_5; \gamma_\alpha, \gamma_\alpha \gamma_5; \sigma_{\alpha\beta}\}$ . Its expansion into local operators reads

$$O_{\Gamma}(0, \kappa x) = \sum_{n=0}^{\infty} \frac{\kappa^n}{n!} x^{\mu_1} x^{\mu_2} \dots x^{\mu_n} \times \quad (3.1)$$

$$[\bar{\psi}(0)\Gamma D_{\mu_1}(y)D_{\mu_2}(y)\dots D_{\mu_n}(y)\psi(y)] \big|_{y=0} ,$$

where  $D_{\mu}(y)$  denotes the covariant derivative taken at  $y$ . Now, the local operators whose tensor structure is determined by  $\Gamma$ , e.g.,  $\alpha\beta(\mu_1 \dots \mu_n)$ , and being totally symmetric with respect to  $\mu_1 \dots \mu_n$  are to be decomposed into irreducible tensors. These tensors should be traceless and their symmetry behavior is uniquely determined by Young patterns  $(m_1, m_2, \dots m_r)$  whose  $i$ -th row has length  $m_i$ . In the (pseudo) scalar case the only allowed Young pattern is  $(n)$ , in the (axial) vector case there are two Young patterns  $(n+1)$  and  $(n, 1)$ , whereas in the antisymmetric resp. symmetric tensor case the Young patterns  $(n+1, 1)$  and  $(n, 1, 1)$  resp.  $(n+2)$ ,  $(n+1, 1)$  and  $(n, 2)$  appear. The complete decomposition of the local tensors into irreducible ones besides the leading twist part contains also irreducible tensors of higher twist being related to the trace terms (e.g., in the symmetric tensor case contributions up to twist 6 occur, cf. [ 6]). The next step to be performed consists in resumming, according to (3.1), the towers (with respect to  $n$ ) of the local operators with the same twist to non-local operators of definite twist which are tensorial harmonic functions. Let us remark that according to this definition the twist decomposition depends on the basic point  $y = 0$  where the local expansion is made. Finally, these non-local operators are to be projected onto the light-cone in order

to obtain the twist decomposition of the light-ray operators we are seeking for.

Now we list those results of [ 5] which are relevant for the present consideration:

(a) Twist-2 scalar quark operators for arbitrary positions  $\kappa_1 x$  and  $\kappa_2 x$  are given by

$$O^{\text{tw}2}(\kappa_1 x, \kappa_2 x) = O(\kappa_1 x, \kappa_2 x) \quad (3.2)$$

$$+ \sum_{k=1}^{\infty} \int_0^1 dt \left(\frac{1-t}{t}\right)^{k-1} \frac{(-x^2)^k \square^k}{4^k k! (k-1)!} O(\kappa_1 t x, \kappa_2 t x),$$

with (compare Eq. (2.2))

$$O(\kappa_1 x, \kappa_2 x) = \quad (3.3)$$

$$= \frac{i}{2} RT \{ [\bar{\psi}(\kappa_1 x)(x\gamma)U(\kappa_1 x, \kappa_2 x)\psi(\kappa_2 x) - \bar{\psi}(\kappa_2 x)(x\gamma)U(\kappa_2 x, \kappa_1 x)\psi(\kappa_1 x)] S \} .$$

The operator (3.2) being of leading twist, i.e., with all traces being subtracted, obeys the following important relation

$$\square O^{\text{tw}2}(\kappa_1 x, \kappa_2 x) = 0 . \quad (3.4)$$

Obviously, on the light-cone,  $x \rightarrow \tilde{x}$ , both operators, (3.2) and (3.3), coincide.

(b) Twist-2 vector quark operators are shown to be determined through the twist-2 scalar quark operator by [ 5]

$$O_{\alpha}^{\text{tw}2}(\kappa_1 x, \kappa_2 x) = \int_0^1 d\tau \partial_{\alpha} O^{\text{tw}2}(\kappa_1 \tau x, \kappa_2 \tau x), \quad (3.5)$$

which, after projecting onto the light-cone, is crucial for the reduction of the matrix elements of the operators (2.4, 2.5) to those of the corresponding (pseudo)scalar operators. The operator (3.5) satisfies the relations

$$\square O_{\alpha}^{\text{tw}2}(\kappa_1 x, \kappa_2 x) = 0 , \quad (3.6)$$

$$\partial^{\alpha} O_{\alpha}^{\text{tw}2}(\kappa_1 x, \kappa_2 x) = 0 . \quad (3.7)$$

The second of these relations is crucial for current conservation. Let us remark that on the light-cone these relations have to be written by using the interior derivative [ 11], namely

$$\partial_{\alpha} \rightarrow d_{\alpha} \equiv (1 + \tilde{x}\tilde{\partial})\tilde{\partial}_{\alpha} - \frac{1}{2}\tilde{x}_{\alpha}\tilde{\partial}^2 \quad \text{with} \quad d^2 = 0 .$$

Quite analogous relations hold for the pseudoscalar as well as axial vector operators.

#### 4. MATRIX ELEMENTS OF TWIST-2 OPERATORS

Our aim is to obtain an expression for the non-forward Compton amplitude. Up to now we considered the representation of the  $T$ -product of currents containing twist-2 non-local (axial) vector operators. The next step will be to perform matrix elements of that  $T$ -product which, because of Eq. (3.5), can be traced back to matrix elements of the (pseudo) scalar operators (3.2). Let us consider these matrix elements first. They decompose into two parts having a Dirac and a Pauli structure, respectively:

$$\begin{aligned} e^2 \langle p_2, S_2 | O^{\text{tw}2}(x/2, -x/2) | p_1, S_1 \rangle & \quad (4.1) \\ = i \bar{u}(p_2, S_2) (\gamma x) u(p_1, S_1) \times \\ & \int Dz e^{-ixp(z)/2} f(z_1, z_2, p_i p_j x^2, p_i p_j, \mu_R^2) \\ + i \bar{u}(p_2, S_2) (x \sigma p_-) u(p_1, S_1) \times \\ & \int Dz e^{-ixp(z)/2} g(z_1, z_2, p_i p_j x^2, p_i p_j, \mu_R^2), \end{aligned}$$

where  $(x \sigma p_-) \equiv x^\alpha \sigma_{\alpha\beta} p_-^\beta$  and  $p(z) \equiv p_1 z_1 + p_2 z_2$ ,  $\mu_R$  denotes the renormalization scale and

$$Dz = \frac{1}{2} dz_1 dz_2 \theta(1-z_1) \theta(1+z_1) \theta(1-z_2) \theta(1+z_2).$$

The kinematic decomposition given above follows if one takes into account that the spinors  $u(p_i, S_i)$  describing the hadrons satisfy the free Dirac equation. The functions  $f(z_1, z_2, (p_i p_j) x^2, (p_i p_j), \mu_R^2)$  are the parton distribution amplitudes and  $z_i$  are the momentum fractions. For brevity we drop the remaining variables. In the present approach we, moreover, set  $(p_i p_j) \approx 0$ . Under these assumptions the relation (3.4) is also valid for the matrix elements:

$$\begin{aligned} \square \langle p_2, S_2 | O^{\text{tw}2}(x/2, -x/2) | p_1, S_1 \rangle & = \quad (4.2) \\ \langle p_2, S_2 | \square O^{\text{tw}2}(x/2, -x/2) | p_1, S_1 \rangle & = 0. \end{aligned}$$

Now, let us reconstruct the vector operator using Eq. (3.5). We obtain

$$\begin{aligned} e^2 \langle p_2, S_2 | O_\alpha^{\text{tw}2}(x/2, -x/2) | p_1, S_1 \rangle & \quad (4.3) \\ = i \int Dz e^{-ixp(z)/2} F(z_1, z_2) \times \\ & [\bar{u}(p_2, S_2) \gamma_\alpha u(p_1, S_1) \end{aligned}$$

$$\begin{aligned} & -\frac{i}{2} p_\alpha(z) \bar{u}(p_2, S_2) (\gamma x) u(p_1, S_1)] \\ + i \int Dz e^{-ixp(z)/2} G(z_1, z_2) \times \\ & [\bar{u}(p_2, S_2) \sigma_{\alpha\beta} p_-^\beta u(p_1, S_1) \\ & -\frac{i}{2} p_\alpha(z) \bar{u}(p_2, S_2) (x \sigma p_-) u(p_1, S_1)], \end{aligned}$$

where

$$F(z_1, z_2) = \int_0^1 \frac{d\lambda}{\lambda^2} f\left(\frac{z_1}{\lambda}, \frac{z_2}{\lambda}\right), \quad (4.4)$$

and an analogous representation connects  $G$  to  $g$ . Moreover, similar representations between the corresponding functions  $f_5$ ,  $F_5$ ,  $g_5$  and  $G_5$  are valid for the operators containing  $\gamma_5$ . Note that also here the necessary conditions

$$\square \langle p_2, S_2 | O_{(5)\alpha}^{\text{tw}2}(x/2, -x/2) | p_1, S_1 \rangle = 0 \quad (4.5)$$

$$\partial^\alpha \langle p_2, S_2 | O_{(5)\alpha}^{\text{tw}2}(x/2, -x/2) | p_1, S_1 \rangle = 0 \quad (4.6)$$

are satisfied. The relations (4.4) are of *central importance* since they form the theoretical basis of the Wandzura–Wilczek relations and allow expressions for the non-forward Compton amplitude based on expectation values of scalar operators only.

#### 5. CURRENT CONSERVATION

Current conservation is a very important criterion for the relevance of the derived expressions. Formally we have to start with Eq. (1.1), and apply Eq. (2.6) and the expressions for the matrix elements (4.3). In the case of forward scattering current conservation holds [12].

For non-forward scattering one is confronted with the following problem. Let us consider the asymmetric current product with respect to  $x = 0$

$$\begin{aligned} \hat{T}_{\mu\nu}^{\text{tw}2}(\kappa_1 x, \kappa_2 x) & = -e^2 \frac{x^\lambda}{i\pi^2(x^2 - i\epsilon)^2} \times \quad (5.1) \\ & [S_{\mu\lambda\nu}^\alpha O_\alpha^{\text{tw}2}(\kappa_1 x, \kappa_2 x) - i\varepsilon_{\mu\lambda\nu}^\alpha O_5^{\text{tw}2}(\kappa_1 x, \kappa_2 x)], \end{aligned}$$

where  $\kappa_1 - \kappa_2 = 1$ . It is easy to convince oneself that, independent of the values of  $\kappa_i$ ,

$$\partial_x^\mu \hat{T}_{\mu\nu}(\kappa_1 x, \kappa_2 x) = 0 = \partial_x^\nu \hat{T}_{\mu\nu}(\kappa_1 x, \kappa_2 x), \quad (5.2)$$

holds because the relations (3.6) and (3.7) are satisfied. These relations, ensuring tracelessness

of the vector operators of definite twist, are not changed by perturbation theory and renormalization.

This proves conservation of the first (or second) current if  $\kappa_1 = 1, \kappa_2 = 0$  (or  $\kappa_1 = 0, \kappa_2 = -1$ ) is chosen. However, if conservation of both electromagnetic currents simultaneously shall be proven we have to study

$$iRT[J_\mu(x)J_\nu(y)S]^{tw2} = -e^2 \frac{\xi^\lambda}{i\pi^2(\xi^2 - i\epsilon)^2} \times \\ [S_{\mu\lambda\nu}^\alpha O_\alpha^{tw2}(\eta + \frac{\xi}{2}, \eta - \frac{\xi}{2}) \\ - i\epsilon_{\mu\lambda\nu}^\alpha O_5^{tw2}(\eta + \frac{\xi}{2}, \eta - \frac{\xi}{2})], \quad (5.3)$$

where  $\eta = (x + y)/2$  by convention denotes the reference point for the twist decomposition, and  $\xi = x - y$  approaches the light-cone. Because of

$$O_{(5)\alpha}^{tw2}(\eta + \frac{\xi}{2}, \eta - \frac{\xi}{2}) := e^{-i\eta P} O_{(5)\alpha}^{tw2}(\frac{\xi}{2}, -\frac{\xi}{2}) e^{i\eta P},$$

where  $P_\mu$  is the momentum operator, we find that Eqs. (5.2) hold with respect to the variable  $\xi$  which is part of  $x$  and  $y$ . Therefore, applying both derivations, either  $\frac{\partial}{\partial x^\mu} = \frac{1}{2}\frac{\partial}{\partial \eta^\mu} + \frac{\partial}{\partial \xi^\mu}$  or  $\frac{\partial}{\partial y^\mu} = \frac{1}{2}\frac{\partial}{\partial \eta^\mu} - \frac{\partial}{\partial \xi^\mu}$ , to the expression (5.3) there remains, in both cases, a non-vanishing part which is proportional to  $[P_\mu, O_{(5)\alpha}^{tw2}(\eta + \frac{\xi}{2}, \eta - \frac{\xi}{2})]$ .

This shows that the proof of current conservation essentially depends on the choice of the reference point for the twist definition: translation of the non-linear operator also shifts that reference point. It prevented us from proving conservation of both currents simultaneously. This intrinsic problem of all the twist definitions could be very important when non-leading twist contributions are considered.

Let us turn to the case of twist 2 now. For the explicit calculations of the resulting expressions we use a normalized helicity basis, cf. [2], with  $\varepsilon_{0\mu}^{(i)} = q_{i\mu}/\sqrt{|q_i^2|}$  for  $q_i^2 < 0$  resp.  $\varepsilon_{0\mu}^{(i)} = q_{i\mu}/(\sqrt{2}|q_{0i}|)$  for  $q_i^2 = 0$ . We have shown in Ref. [2], that the current violating contributions are of  $O(\nu^{-1/2} \times \text{OME})$  or higher order. These terms are of higher twist and have to be dealt with the operator matrix elements of the higher twist operators.<sup>2</sup>

<sup>2</sup>C. Weiss has proven recently [13] using the representation of [10] that the terms  $\propto O(\nu^{-1/2})$  cancel with corresponding terms due to twist-3 operators.

## 6. INTEGRAL RELATIONS

Here we present the final expression for the Compton amplitude. The calculations are given in [2]. We split the amplitude into its symmetric part and the antisymmetric part  $T^{\mu\nu} = T_s^{\mu\nu} + T_{as}^{\mu\nu}$ . In the case of forward scattering the former one corresponds to the unpolarized and the latter one to the polarized contribution. First we consider the symmetric part:

$$T_s^{\mu\nu, tw2} = -\frac{1}{\nu} \int_{-1}^1 dt \frac{1}{\xi + t - i\epsilon} \times \quad (6.1) \\ \left\{ \left[ 2(g^{\mu\nu}(qp_+) - (q^\mu p_+^\nu + q^\nu p_+^\mu)) f_1(t, \eta) \right. \right. \\ \left. \left. + p_+^\mu p_+^\nu f_2(t, \eta) \right] \bar{u}(p_2, S_2)(\gamma q)u(p_1, S_1) \right. \\ \left. + \left[ 2(g^{\mu\nu}(qp_+) - (q^\mu p_+^\nu + q^\nu p_+^\mu)) g_1(t, \eta) \right. \right. \\ \left. \left. + p_+^\mu p_+^\nu g_2(t, \eta) \right] \bar{u}(p_2, S_2)(q\sigma p_-)u(p_1, S_1) \right\} \\ + \text{non-leading terms},$$

where  $t = z_+ + \eta z_-$ ,  $z_\pm = \frac{1}{2}(z_2 \pm z_1)$ . Here the partition functions  $f_i$  and  $g_i$  are ‘one-variable’ distribution amplitudes which are defined by

$$f_{(5)}(t, \eta) = \int dz_- f_{(5)}(z_+ = t - \eta z_-, z_-), \quad (6.2)$$

$$g_{(5)}(t, \eta) = \int dz_- g_{(5)}(z_+ = t - \eta z_-, z_-), \quad (6.3)$$

from the ‘two-variable’ distribution amplitudes used in the representation (4.1) of the matrix elements of the (pseudo) scalar operators. Unlike the case of forward scattering these functions do not depend on scaling variables only but besides of the scaling variable  $\eta$  which describes non-forwardness of the combination of momentum fractions  $t$ . The following new relations are obtained between the amplitude-functions  $f_i(g_i)$ , see [2]:

$$f_2(t, \eta) = 2t f_1(t, \eta) \equiv 2f(t, \eta), \quad (6.4)$$

$$g_2(t, \eta) = 2t g_1(t, \eta) \equiv 2g(t, \eta). \quad (6.5)$$

These relations are structurally similar to the CALLAN-GROSS relation for forward scattering. There, by virtue of the optical theorem,  $\frac{1}{\xi + t + i\epsilon} \rightarrow i\pi\delta(t + \xi)$  and for  $p_2 \rightarrow p_1 = p$  it follows  $t \rightarrow z_+ \equiv$

$z, t$  is turned into a scaling variable. In the above expressions furthermore

$$\begin{aligned}\bar{u}(p_2, S_2)(\gamma q)u(p_1, S_1) &\rightarrow 2pq, \\ \bar{u}(p_2, S_2)(q\sigma p_-)u(p_1, S_1) &\rightarrow 0\end{aligned}$$

holds in the latter case.

The result for the antisymmetric part is more complicated:

$$\begin{aligned}T_{(\lambda_1)(\lambda_2)} &= \epsilon_{\mu(\lambda_1)}^{(2)} \epsilon_{\nu(\lambda_2)}^{(1)} T_{\text{as}}^{\mu\nu, \text{tw}2} \\ &= i\epsilon^{\mu\rho\nu\sigma} \epsilon_{\mu(\lambda_1)}^{(2)} \epsilon_{\nu(\lambda_2)}^{(1)} B_{\rho\sigma}\end{aligned}\quad (6.6)$$

with

$$\begin{aligned}B^{\rho\sigma} &= -\frac{q^\rho}{\nu^2} \left[ \int_{-1}^1 dt \frac{1}{\xi + t - i\varepsilon} \times \right. \\ &\quad \left\{ \left( f_{5,1}(t, \eta) + f_{5,2}(t, \eta) \right) \nu S_\sigma^{12} \right. \\ &\quad \left. + \left( g_{5,1}(t, \eta) + g_{5,2}(t, \eta) \right) \nu \Sigma_\sigma^{12} \right. \\ &\quad \left. + f_{5,2}(t, \eta) p_+^\sigma (q S^{12}) + g_{5,2}(t, \eta) p_+^\sigma (q \Sigma^{12}) \right\} \Big].\end{aligned}$$

Here, we used the following abbreviations for the spinor structure

$$\begin{aligned}S_\sigma^{12} &= -\frac{1}{2} \bar{u}(p_2, S_2) \gamma_5 \gamma_\sigma u(p_1, S_1), \\ \Sigma_\sigma^{12} &= -\frac{1}{2} \bar{u}(p_2, S_2) \gamma_5 \sigma_{\sigma\rho} p_-^\rho u(p_1, S_1).\end{aligned}$$

In the case of forward scattering  $p_2 \rightarrow p_1 = p$ ,  $S_\sigma^{12} \rightarrow S_\sigma$  and  $\Sigma_\sigma^{12} \rightarrow 0$  holds, where  $S_\sigma$  is the spin vector introduced for forward scattering. We obtain the following relations for the amplitude functions, cf. [ 2 ] :

$$f_{5,1}(t, \eta) \equiv f_5(t, \eta), \quad (6.7)$$

$$f_{5,2}(t, \eta) = -f_5(t, \eta) + \int_t^{\text{sgn } t} dz \frac{f_5(z, \eta)}{z}, \quad (6.8)$$

$$g_{5,1}(t, \eta) \equiv g_5(t, \eta), \quad (6.9)$$

$$g_{5,2}(t, \eta) = -g_5(t, \eta) + \int_t^{\text{sgn } t} dz \frac{g_5(z, \eta)}{z}. \quad (6.10)$$

Again  $f_{5,i}$  and  $g_{5,i}$  depend on the momentum fraction  $t$  and a scaling variable. These relations generalize the WANDZURA–WILCZEK relation of deep inelastic scattering to non-forward amplitudes.

## 7. CONCLUSIONS

We studied the structure of the virtual Compton amplitude for deep-inelastic non-forward scattering  $\gamma^* + p \rightarrow \gamma'^* + p'$  in lowest order in QED in the massless limit. In the generalized Bjorken region ( $qp_+$ ),  $-q^2 \rightarrow \infty$  the twist-2 contributions to the Compton amplitude were calculated using the non-local operator product expansion. The twist separation and the relations between twist-2 vector operators and twist 2 scalar operators are essential for the current conservation at the level of twist-2 and, moreover, all parton distributions are connected with the matrix elements of the scalar twist-2 operators.

The relations between the twist-2 contributions of the unpolarized and polarized amplitude functions were derived. They are the non-forward generalizations of the CALLAN–GROSS and WANDZURA–WILCZEK relations for unpolarized and polarized deep-inelastic forward scattering. The relations for the DIRAC and PAULI parts are of the same form.

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